Dust acoustic solitons with variable particle charge: Role of the ion distribution

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Dust-acoustic solitons of large amplitude with variable particle charge are studied using the Sagdeev quasipotential analysis. Two limiting cases of ion distribution are considered separately: Boltzmann and highly energetic cold ions. It is shown that in both cases only compressive (density) solitons are possible. The charge variation is not important in rarefied particle clouds, but becomes crucial if the particle number density is sufficiently high. Analytical expressions for the range of Mach numbers where solitons might exist are obtained. It is found that solitons are allowed in the supersonic regime, and that in dense clouds the width of the Mach number range remains finite for the Boltzmann ions, but tends to zero for highly energetic ions.

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Wave motion of charged micron-sized particles in a plasma—the so-called dust-acoustic (DA) mode [1]—has been studied extensively. Most of the work was focused on the investigation of small-amplitude waves exploring the influence of various parameters on properties of the wave dispersion relation. Large-amplitude (nonlinear) stationary DA waves might also exist in complex (dusty) plasmas: Similar to the ion-acoustic (IA) waves, nonlinear corrections to the DA phase velocity makes the wave front steeper, whereas the dispersion at short wavelengths has the opposite effect [2]. Therefore, a balance between these two mechanisms is possible, leading to stationary nonlinear DA waves.

One of the most interesting types of nonlinear waves are solitary waves. DA solitons of large amplitude have been studied in a number of papers, using the Sagdeev quasipotentials [1,3–9]. It was shown that particular ion distributions are required for the existence of both compressive and rarefactive solitons [6], as well as for double layers [4,6]. It was also emphasized that the self-consistent variation of the particle charge in the wave might be important [7–9], and distinguishes the DA solitons qualitatively from the IA solitons.

In this paper we study the Sagdeev quasi-potential for DA solitons with variable particle charge, and we consider separately two limiting cases of the ion distribution: Boltzmann and highly energetic cold ions. This allows us to understand qualitatively how the soliton solution depends on the possible plasma parameters in a discharge (bulk plasma, sheath). In particular, we show that the charge variation is not important in rarefied particle clouds, but becomes crucial if the particle number density is sufficiently high. We obtain analytical expressions for the range of Mach numbers where solitons might exist, and the corresponding values of the electric potential in the wave. The derived scaling dependencies could be useful for comparison with experimental measurements.

We assume that the thermal velocity of the particles is much smaller than the phase velocity of DA waves. This is valid when the the ratio of the particle kinetic temperature to the ion temperature is much less than the particle charge number: $T/T_i \ll Z$ (usually, $T/T_i \lesssim 10^2$ and $Z \sim 10^3 - 10^4$). Then we can neglect both the particle pressure and kinetic effects caused by the particle-wave interaction [10], and use the fluid equations for the particle component,

$$\frac{\partial n}{\partial t}+\frac{\partial n v}{\partial z}=0,$$
$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial z}=\frac{Ze}{m}\frac{\partial \phi}{\partial z}, \tag{1}$$

where m and n are the particle mass and number density, v is the particle velocity, and ϕ is the electric potential in the wave which is described by the Poisson equation,

$$\frac{\partial^2 \phi}{\partial z^2}=-4\pi e(n_i-n_e-Zn). \tag{2}$$

Here n_e and n_i are the electron and ion number densities. The time scale of DA wave is $\gtrsim \omega_{\rm pd}^{-1} \sim 1-10^{-2}$ s, so that it is reasonable to suppose that the Maxwellization time for electrons is much smaller. Thus, we can use the Boltzmann distribution for electrons,

$$n_e = n_{e0}\exp\left(\frac{e\phi}{T_e}\right). \tag{3}$$

Note that the smallness of the thermal particle velocity allows us to neglect the bulk viscosity term (due to interparticle collisions) in the momentum equation (1). The friction term due to collisions with atoms of neutral gas is omitted as well. The applicability of this approach is discussed later.

The particle charge variation in the wave is governed by the kinetic charging equation. However, for real experimental conditions we do not need to solve this equation, because the typical charging time of particles, $\sim 10^{-6}-10^{-3}$ s, is much smaller than the time scale of the wave. Therefore, we can expect that the charge is always close to the equilibrium value given by the balance of the electron and ion fluxes on the particle surface,

$$J_e - J_i \simeq 0. \tag{4}$$

The fluxes depend on the charge number Z and the local plasma density. In turn, the latter is determined by the local potential ϕ, i.e., Eq. (4) is an implicit relation between Z and ϕ. The electron flux for the Maxwell distributed electrons is [11]

$$J_e(\phi,Z)=2\sqrt{2\pi}a^2 n_{e0} v_{T_e} e^{\Phi} e^{-\gamma}, \tag{5}$$

where a is the particle radius, $v_{T_e}=\sqrt{T_e/m_e}$ is the electron thermal velocity, $\Phi(\phi)=e\phi/T_e$ is the dimensionless electric potential and $\gamma(Z)=e^2Z/aT_e$ is the normalized charge number. Note that the value of γ depends on the type of gas in the discharge, as well as on the discharge conditions, but is always of the order of a few [12]. In the absence of waves, the (undisturbed) densities and the charge number should obey the quasineutrality condition,

$$n_{i0}=n_{e0}+Z_0 n_0.$$

In a nonlinear stationary wave all the variables depend on the self-similar combination

$$\xi=z-ut, \tag{6}$$

where u is the wave velocity. As mentioned before, we consider two limiting cases for the ion distribution function: the Boltzmann distribution, which is reasonable for an isotropic bulk plasma (where the rapid Maxwellization is provided by ion-neutral collisions) and highly energetic ions with flow velocity much higher than the IA velocity (which corresponds to a region inside the plasma sheath). For a particle cloud suspended in the pre-sheath region we would presumably have some ''intermediate'' situation.

Boltzmann-distributed ions. The ion density obeys the relation

$$n_i=n_{i0}\exp\left(-\frac{e\phi}{T_i}\right). \tag{7}$$

The corresponding flux on the particle surface is [11]

$$J_i(\phi,Z)=2\sqrt{2\pi}a^2 n_{i0} v_{T_i} e^{-\tau\Phi}(1+\tau\gamma), \tag{8}$$

where $\tau=T_e/T_i\sim 30-100$ for typical discharge conditions. Substituting Eqs. (5) and (8) in Eq. (4) and neglecting terms $O(\tau^{-1})$ we obtain the relation between Φ and γ,

$$\gamma-\gamma_0+\ln(\gamma/\gamma_0)\simeq\tau\Phi. \tag{9}$$

This is a transcendental equation with respect to γ, but it can be solved approximately assuming that the relative variation of the charge is sufficiently small (or γ/γ_0 close to 1, we will return to this later). Then, expanding $\ln(\gamma/\gamma_0)\simeq(\gamma-\gamma_0)/\gamma_0$ in Eq. (9) we obtain $(1+\gamma_0^{-1})(\gamma-\gamma_0)\simeq\tau\Phi$, or for the charge number

$$Z(\Phi)\simeq Z_0\left(1+\frac{\tau\Phi}{1+\gamma_0}\right). \tag{10}$$

Using Eq. (10), we can integrate Eq. (1) for the self-similar variable (6) and obtain the density

$$n=n_0\left[1+\frac{2}{M_*^2}\left(\Phi+\frac{\tau\Phi^2}{2(1+\gamma_0)}\right)\right]^{-1/2}, \tag{11}$$

where $M_*^2=mu^2/Z_0T_e$. Then, substituting Eqs. (3), (7), (10), and (11) in the Poisson equation (2) and integrating, we derive the ''energy integral''

$$\tfrac{1}{2}\lambda_{De}^2(\Phi'_\xi)^2=\mathcal{E}-U(\Phi), \tag{12}$$

where λ_{De} is the electron Debye length, $\lambda_{De}^{-2}=4\pi e^2 n_{e0}/T_e$, and $\mathcal{E}$ is the ''total energy of the oscillator'' with the Sagdeev pseudopotential

$$-U(\Phi)=(1+P)\tau^{-1}(e^{-\tau\Phi}-1)+e^{\Phi}-1+PM_*^2\left[\sqrt{1+\frac{2}{M_*^2}\left(\Phi+\frac{\tau\Phi^2}{2(1+\gamma_0)}\right)}-1\right]. \tag{13}$$

Here $P=Z_0n_0/n_{e0}$ is the Havnes parameter which is a measure of the volume particle charge.

The pseudopotential (13) tends to zero at $\Phi\to 0$. Then the localized soliton solution of Eq. (12) can exist for $\mathcal{E}=0$ if (i) $U'|_{\Phi=0}=0$, (ii) $U(\Phi)$ is a potential well, and (iii) $U(\Phi_{\max})=0$ for some finite $\Phi_{\max}$ [but $U'(\Phi_{\max})\neq 0$]. The first condition is satisfied identically. The second one requires $U''|_{\Phi=0}<0$. Expanding Eq. (13) we find that condition (ii) is satisfied if $M>1$, where $M=u/C_{\mathrm{DA}}$ is the Mach number (note that $M_*\propto M$) normalized to the phase velocity of the DA waves with variable particle charge [13],

$$C_{\mathrm{DA}}=\sqrt{\frac{1+\gamma_0}{1+\gamma_0+\frac{P}{1+P}}}\sqrt{\frac{Z_0P}{1+P}\frac{T_i}{m}}, \tag{14}$$

[when the particle fraction is small, $P\ll 1$, or the charge is large, $\gamma_0\gg 1$, the influence of the charge variation on the phase velocity vanishes—the first factor in Eq. (14) tends to unity]. Thus we get the natural result that the DA solitons can exist only in the supersonic regime. The requirement (iii) (which is the sufficient condition for solitons to exist) determines the maximum value of the potential, $\Phi_{\max}$, in the wave. For weakly supersonic solitons, $0<M-1\ll 1$, we obtain from Eq. (13) the single root $\Phi_{\max}(M)$,

$$-\tau\Phi_{\max}\sim\frac{2P}{1+P}(M-1),$$

which means that for $M\gtrsim 1$ there only compressive density solitons exist [see Eq. (11)]. The absolute value of $\Phi_{\max}$ increases monotonically as the Mach number grows until the argument of the square root in Eq. (13) equals zero. The corresponding critical value of the potential, Φ_{cr}, is the smaller root of the quadratic equation

$$(1+\gamma_0)^{-1}(\tau|\Phi_{\mathrm{cr}}|)^2-2\tau|\Phi_{\mathrm{cr}}|+\tau M_*^2=0, \tag{15}$$

and the corresponding critical value of the Mach number is $M_{\rm cr}=M(\Phi_{\rm cr})$. Above $M_{\rm cr}$ there is no solution. Physically this limit arises because the travelling potential barrier, $Z(\Phi)\Phi$, becomes too high—the particles cannot get across it and are reflected by the wave front. The reflected precurser flux upstream of the soliton leads to the formation of a shock wave [2]. [Note that if $\tau M_*^2>(1+\gamma_0)$ then Eq. (15) has no real roots and the argument of the square root in Eq. (13) is always positive, because the charge decreases too fast, see Eq. (10). However, equation $U(\Phi_{\rm max})=0$ has no solutions in this case].

In order to determine the critical value $M_{\rm cr}$ (and $\Phi_{\rm cr}$) let us assume that $|\Phi_{\rm cr}|\ll 1$ [below we show that $\Phi_{\rm cr}\sim O(\tau^{-1})$], so that one can expand $e^{\Phi}\simeq 1+\Phi$ in Eq. (13). Then $U(\Phi_{\rm cr})=0$ yields

$$\tau|\Phi_{\rm cr}|+P\tau M_*^2\simeq(1+P)(e^{\tau|\Phi_{\rm cr}|}-1).$$

Substituting $M_*^2(\Phi_{\rm cr})$ from Eq. (15) we obtain the equation for $\Phi_{\rm cr}$,

$$\frac{1+2P}{1+P}\tau|\Phi_{\rm cr}|-\frac{P(\tau|\Phi_{\rm cr}|)^2}{(1+P)(1+\gamma_0)}\simeq e^{\tau|\Phi_{\rm cr}|}-1. \qquad (16)$$

Equation (16) can be solved approximately in the limits of rarefied ($P\ll 1$) and dense ($P\gg 1$) particle clouds,

$$\begin{aligned} P\ll 1:&\quad \tau|\Phi_{\rm cr}|\simeq 2P,\\ P\gg 1:&\quad \tau|\Phi_{\rm cr}|\simeq\frac{2+\gamma_0}{(e-2)\gamma_0+e}, \end{aligned} \qquad (17)$$

where $e=2.71\ldots$ [relative error of solution (17) for $P\gg 1$ is $\lesssim 5\%$]. Using Eq. (10) we see that for reasonable conditions ($\gamma_0\gtrsim 2$) the maximum possible value of the relative charge variation is $\lesssim 0.3$, i.e., the linear expansion of the logarithm in Eq. (9) is justified. Substituting Eq. (17) in Eq. (15) we determine $M_*(\Phi_{\rm cr})$, and using the relation

$$M^2=\left(\frac{1}{1+\gamma_0}+\frac{1+P}{P}\right)\tau M_*^2,$$

we finally obtain the critical Mach number,

$$P\ll 1:\quad M_{\rm cr}\simeq 2,$$

$$P\gg 1: \qquad (18)$$

$$M_{\rm cr}\simeq\left(\frac{2+\gamma_0}{1+\gamma_0}\right)\frac{\sqrt{2(e-2)\gamma_0^2+(4e-5)\gamma_0+2(e-1)}}{(e-2)\gamma_0+e}.$$

Thus in rarefied clouds the variation of the potential (17) is weak (due to the small particle volume charge); $M_{\rm cr}\simeq 2$ and does not depend on γ_0, in fact it equals the value obtained without charge variations. In the opposite case of dense clouds, $M_{\rm cr}$ is a function of γ_0, i.e., the charge variation is important. Figure 1 shows the dependence of $M_{\rm cr}$ on γ_0 for $P\gg 1$. For realistic conditions, the value of γ_0 varies from $\simeq 1.5$ to $\simeq 6$ [12]. (Note that if we neglect the charge variation in the above calculations, which formally corresponds to the limit $\gamma_0\to\infty$, then $M_{\rm cr}\simeq 1.58$ for $P\gg 1$. This coincides with the value obtained in [5]). We see that the variable charge narrows the range of Mach numbers, $M_{\rm cr}-1$, where solitons can exist.

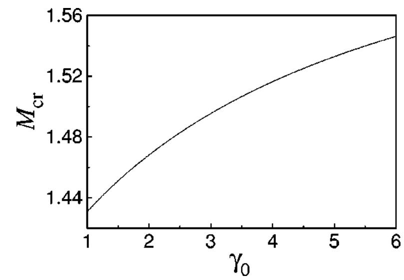

FIG. 1. The upper Mach number limit (18) for the soliton solution, $M_{\rm cr}$, in a dense particle cloud ($P\gg 1$) versus the dimensionless particle charge, γ_0.

Highly energetic ions. Now we consider the case when the ion drift velocity, V_i, exceeds the velocity of IA waves, $V_i\gg\sqrt{T_e/m_i}$, and when the spread of the velocity distribution is much less than V_i. Since the scale of the potential variation in the wave is $\Phi\lesssim 1$, the corresponding variation of the drift velocity should be relatively small. Hence, we can assume homogeneous ion density,

$$n_i\simeq n_{i0}, \qquad (19)$$

which implies that ions do not participate in collective processes (screening, waves, etc.). In this limit the cross section for ion absorption on a particle is approximately πa^2, and the ion flux is

$$J_i(\phi,Z)\simeq\pi a^2 n_{i0}V_i. \qquad (20)$$

Substituting Eq. (20) together with Eq. (5) in Eq. (4) we have $\gamma=\gamma_0+\Phi$, or for the charge number,

$$Z(\phi)=Z_0(1+\gamma_0^{-1}\Phi). \qquad (21)$$

Using Eqs. (19) and (21), we derive from Eqs. (1) and (2) the Sagdeev potential,

$$\begin{aligned}-U(\Phi)=&-(1+P)\Phi+e^{\Phi}-1\\ &+PM_*^2\left[\sqrt{1+\frac{2}{M_*^2}\left(\Phi+\frac{\Phi^2}{2\gamma_0}\right)}-1\right].\end{aligned} \qquad (22)$$

The condition $U''|_{\Phi=0}<0$ requires $M>1$, where the Mach number $M=u/\tilde{C}_{\rm DA}$ now is normalized to the DA phase velocity without ion screening,

$$\tilde{C}_{\rm DA}=\frac{1}{\sqrt{1+P/\gamma_0}}\sqrt{Z_0P\frac{T_e}{m}}.$$

Hence only compressive supersonic solitons are possible for the pseudo-potential (22). This is similar to the case of Boltzmann ions. The upper Mach number limit, $M_{\rm cr}$, of the solution is determined by the argument of the square root in Eq. (22). The critical potential is given by the smaller root of the quadratic equation

$$\gamma_0^{-1}|\Phi_{\rm cr}|^2-2|\Phi_{\rm cr}|+M_*^2=0, \tag{23}$$

and $M_{\rm cr}=M(\Phi_{\rm cr})$ [the argument of the square root in Eq. (22) is always positive if $M_*^2>\gamma_0$, but then $U(\Phi_{\max})=0$ has no solution]. In order to find the critical Mach number we start by determining $\Phi_{\rm cr}$. Equation $U(\Phi_{\rm cr})=0$ yields

$$(1+P)|\Phi_{\rm cr}|-PM_*^2=1-e^{-|\Phi_{\rm cr}|}.$$

Substituting $M_*^2(\Phi_{\rm cr})$ from Eq. (23) we get for $\Phi_{\rm cr}$,

$$(1-P)|\Phi_{\rm cr}|+\gamma_0^{-1}P|\Phi_{\rm cr}|^2=1-e^{-|\Phi_{\rm cr}|}. \tag{24}$$

One can derive an approximate solution of Eq. (24), which has correct asymptotics for both rarefied and dense particle clouds (when γ_0 is finite),

$$|\Phi_{\rm cr}|\simeq\left(\frac{1}{\gamma_0}+\frac{1}{2P}\right)^{-1}. \tag{25}$$

Using the relation between M and M_*,

$$M^2=\left(\frac{1}{\gamma_0}+\frac{1}{P}\right)M_*^2$$

we obtain the critical Mach number from Eq. (23),

$$M_{\rm cr}\simeq1+\frac{1}{1+2P/\gamma_0}. \tag{26}$$

For $P\ll1$ we get $\Phi_{\rm cr}\simeq2P$ and $M_{\rm cr}\simeq2$. Hence, for rarefied clouds $M_{\rm cr}$ is the same as that in the case of Boltzmann ions, whereas $\Phi_{\rm cr}$ is τ times greater (since ions do not participate in the screening). The charge variation is not important. For dense clouds $M_{\rm cr}$ tends asymptotically to unity as $P\to\infty$. This is very different from the case of Boltzmann ions, where the range $]1,M_{\rm cr}[$ remains finite for any P. Thus, for the energetic streaming ion model [see Eq. (19)] the soliton solution is practically forbidden in sufficiently dense clouds. This strong difference between the two ion models (at $P\gg1$) is solely due to the charge variation effect. Indeed, if we consider particles with constant charge (formally, we take $\gamma_0\to\infty$) with the ion distribution of Eq. (19), then from Eq. (22) we have $|\Phi_{\rm cr}|=M_*^2/2$. In this case equation $U(|\Phi_{\max}|)=0$ has a solution (single root) $|\Phi_{\max}|<|\Phi_{\rm cr}|$ for any M_*^2 $(M^2>1)$, if $P>1$. Hence, for $P>1$ solitons without charge variation are allowed for arbitrary $M>1$ [this can be seen directly also from Eq. (24) which has no solution at $\gamma_0\to\infty$, if $P>1$]. Physically this is because the ion density (19) remains constant inside a soliton (instead of the exponential increase in the case of the Boltzmann distribution). Therefore the term corresponding to the particle density [square root in Eq. (22)] can compensate the ion term for arbitrary large M, if a particle cloud is sufficiently dense $(P>1)$. But when the charge variation is taken into account, it provides an additional correction to the particle density (decreasing it), and thus the ion term in Eq. (22) can be balanced at rather small M only.

Note that even if we would suppose that there is a weak dependence of the ion density on Φ instead of Eq. (19), say $n_i\simeq n_{i0}(1+\epsilon\Phi)$ (where $\epsilon\sim T_e/m_iV_i^2\ll1$), it does not qualitatively change the final expressions for the critical potential (25) and the Mach number (26). Such a linear variation in n_i yields an additional quadratic term, $\frac{1}{2}(1+P)\epsilon|\Phi_{\rm cr}|^2$, in Eq. (24), and if $|\epsilon|\lesssim\gamma_0^{-1}$, the equation is not changed functionally and Eqs. (25) and (26) are still valid for any P.

Discussion and conclusions. Qualitatively, the soliton solution for the DA waves with variable particle charge is similar to that for usual IA waves in the absence of particles. The physical mechanism of the steady-state nonlinear wave formation is a balance between the nonlinear increase of the phase velocity and the dispersion effects which slow down the wave steepening. The soliton solution is possible in the supersonic regime only, but there exists an upper limit of possible Mach numbers, $1<M<M_{\rm cr}$. Above $M_{\rm cr}$ the potential of the wave becomes too high and particles remain in the frame of the wave, braking the soliton. If the particle fraction is low, the variable charge does not affect the characteristics of the DA soliton. The influence of the charge variation on the upper limit of the Mach numbers becomes crucial for sufficiently dense particle clouds, decreasing the value of $M_{\rm cr}$. It is noteworthy that the width of the ''allowed'' Mach number range, $M_{\rm cr}-1$, depends on the ion distribution: The range remains finite for Boltzmann ions, but tends to zero for highly energetic streaming ions. Especially remarkable is that in the absence of charge variation the model of highly energetic streaming ions does not have an upper limit of Mach numbers and solitons are allowed for any $M>1$.

Symmetrical ''pure'' solitons are impossible in real experiments because of dissipation [2]. Along with the usual (collisional) mechanisms of dissipation—particle-neutral friction, viscosity, etc., there could also be a so-called ''collisionless dissipation'': Particles have some finite spread in thermal energy and thus even for allowed Mach numbers there exist some fraction of particles which are reflected upstream by the potential barrier of the wave. (This is similar to cosmic ray shock acceleration [15]). Formally it implies that the ''total energy of oscillator'' $\mathcal{E}$ in Eq. (12) is no longer conserved [14], i.e., the oscillator has lost energy. All these mechanisms should result in shock wave formation, with an oscillatory structure behind the front due to oscillations around the minimum of the Sagdeev pseudo-potential [2]. However, dissipation might be sufficiently weak in experiments where the neutral gas pressure is low. For example, the neutral damping coefficient for ~10 μm particles at pressures ~1 Pa is $\beta\sim0.1-0.3$ s^{-1}. A typical value of the soliton velocity is $u\gtrsim C_{\rm DA}\sim3$–10 cm/s. Therefore, considerable loss of soliton energy occurs over a scale length $C_{\rm DA}/\beta\sim10$–30 cm (or more), which is somewhat larger than the maximum possible size of typical particle clouds in experiments. Hence, we can expect that the transition of a soliton into a shock wave due to neutral gas friction is rather slow for low pressures. The role of the particle thermal spread might also be weak. For reasonable estimates of the thermal velocity, $v_{T_d} \lesssim 0.3$ cm/s, the fraction of the reflected particles is very small, $\lesssim \exp(-C_{\mathrm{DA}}^2/v_{T_d}^2) \sim 10^{-2}$.

And finally, a few general remarks regarding the applicability of the obtained results: In experiments under gravity the particle cloud is normally suspended in the vicinity of the sheath edge, or below it (especially, for large particles). Therefore the model of highly energetic ions is more reasonable for these conditions. Possible experiments under microgravity might be performed in ''nearly'' isotropic plasmas, when the Boltzmann model is a better approach for ions. The role of charge variation is determined by the shape and size of the particle cloud. If the cloud occupies a considerable volume (much larger than the Debye length), then the described charge variation effects should be crucial. However, for thin ''two-dimensional'' clouds the contribution due to variable charge might be rather weak.

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